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Observations on the transient nature of shape-tilting bodies sedimenting in polymeric liquids

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Abstract

The terminal orientation of cylinders with flat ends, sedimenting in polymer solutions are seen to occur with their edges aligned with the direction of fall. This is referred to as shape-tilting in the literature. We show that for cylinders with aspect ratios $L/d \neq 1$, the shape-tilting phenomenon is in fact a transient phenomenon by extending our observation times to up to several hours when compared to a few minutes of observation time in previous experiments. Additionally, our experimental observations are confirmed by use of a simple analytical argument which invokes the symmetries of a Stokes flow and of the sedimenting body.

Keywords: Shape-tilting; Sedimentation; Orientation

1. Introduction

The terminal orientation behavior of rigid bodies in viscoelastic liquids is a curious phenomenon, where the sedimenting bodies assume orientations depending upon the geometry of the body and also upon the nature of the surrounding liquid medium, the two factors being independent of each other. The subject dates back to Thomas and Tait (1879) and Kirchhoff (1869) [also see Lamb (1932) for a detailed discussion on this topic], and has received recent attention (Chiba et al., 1986; Cho et al., 1992; Liu and Joseph, 1993; Joseph and Liu, 1993; Leal, 1975; Galdi and Vaidya, 2001; Galdi et al., 2002; Galdi, 2003; Vaidya, 2004b, 2005). The contribution of the liquid to the orientation behavior is now well understood, however, the geometry of the body as a probable cause requires more serious investigation. Previous observations on this subject (Joseph and Liu, 1993; Wang et al., 2004) claim that cylinders with flat ends, have a tendency to fall, in their steady states, with their edges aligned along the direction of fall (see Fig. 1). Such an orientation is referred to as *shape-tilting*. Experiments performed with cylinders of varying aspect ratios seem to indicate that the shape-tilt is directly proportional to the aspect ratio (namely, ratio of length to diameter) of the cylinder. It is least pronounced in case of low aspect ratios (long cylinders) and becomes more pronounced as the aspect ratio increases (short cylinders). Note that the shape-tilt orientation for flat cylinders and stable steady vertical orientation of ellipsoidal-shaped bodies or round-ended cylinders (Joseph and Liu, 1993) seems to suggest that all rigid bodies have a tendency to fall along their longest axes. However, our experiments with several flat-ended cylinders, in the current study indicates otherwise. Our detailed and systematic experiments in a highly viscous liquid permit us the

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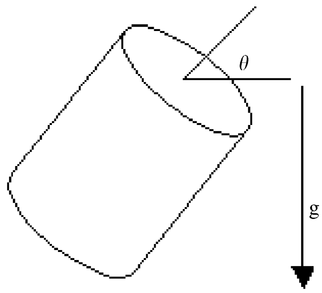

Fig. 1. Shape-tilting.

advantage of very long observation times, at times extending to several hours. Over this time, we note that in most cases, as long as the length to diameter ratio of the particle is not 1, the sedimenting cylinder eventually turns to align its rotational axis either perpendicular to or along the direction of gravity and not its longest axis, implying that, for these cylinders, shape-tilting is merely a transient phenomenon.

2. Experimental set-up

The set-up consists of a sedimentation tank of width 5.5 in, breadth 5.5 in and height 3 feet (1 in = 25.4 mm; 1 ft = 12 in). The dimensions of the tank are chosen to be similar to previous experiments. The experiments are recorded using a digital camera (Cannon XL1) which is placed on a traversing system. The camera can be moved up or down on this system using a motor. The camera is in turn connected to a computer. A live feed of the experiment is captured on the computer which allows us to record the experiment and make accurate measurements of the speed of fall and terminal orientation of the sedimenting body. The sedimentation experiment involved filling the tank with a liquid and dropping a rigid body in it. The liquid filled almost the entire tank and the particle was dropped from rest.

The shape of the sedimenting body was either a prolate spheroid (E), a flat-ended cylinder (CF) or a round-ended cylinder (CR). The dimension and material of the particles were varied in order to change the rate of fall of the body in the liquids. Table 1 lists the dimensions and materials of the particles used in the experiments. Note that in the final column of the table, we define $\gamma = L/d$.

We use a 0.56% concentration of Polyacrylamide (sample AN934SH, SNF Inc.) in water. The rheological properties of the liquid were determined using a cone and plate rheometer (with a 4°, 20 mm steel cone). We summarize these properties in Table 2 based upon the steady flow and oscillatory shear experiments, from which we infer that the liquid possesses very strong viscoelastic and shear-thinning properties. The viscosity, η_0 mentioned in Table 2, refers to the zero shear-rate viscosity which is obtained from fitting the experimental data with the Cross model, while the relaxation time is obtained from the oscillatory shear experiments [Bird and Armstrong (1987), Macosko (1994) and references therein]. Note that the polymer sample used is of considerably high molecular weight compared to any previous liquids used in past sedimentation experiments of this nature (Joseph and Liu, 1993; Liu and Joseph, 1993; Wang et al., 2004).

3. Observations

The experiment involved dropping the different particles in the sedimentation tank from rest. Snapshots of the particles were taken at regular time intervals from which the position and orientation were determined. The observations of the experiment, which was repeated twice for each particle, are recorded in Table 3 and also partly in the Fig. 2. Note that though the height of the liquid in the sedimentation tank remained the same, we indicate different heights in the third column of Table 3. These represent the observed distances traversed by the particles. Though the objective of this paper is primarily, the understanding of the shape-tilting phenomenon in cylinders with flat ends, we also consider a few cases with round-ended cylinders and prolate ellipsoids (see Table 1) to show the marked difference in the behavior of these particles. Our observations may be divided into four categories depending upon the shape of the particles and in the variation of the value of the parameter γ.

The first observation involves the particles E1, E2 and CR1, which have rounded ends; all turn to orient their longest axis parallel to the direction of gravity, in the steady state. However, the behavior of the flat cylinders is not as simple.

Table 1
Particles used in sedimentation experiments

Number	Name	Material	L (in)	d (in)	Density (g/cm^3)	γ
1	E1	Si40	1.0	0.5	1.1	2.0
2	E2	Aluminum	1.0	0.5	2.70	2.0
3	CR1	Teflon	1.0	0.2	2.18	5.0
4	CF1	PET	0.50	0.5	1.37	1.0
5	CF2	PET	0.625	0.5	1.37	1.25
6	CF3	PET	0.75	0.5	1.37	1.50
7	CF4	PET	0.875	0.5	1.37	1.75
8	CF5	PET	1.0	0.5	1.37	2.0
9	CF6	Delrin	0.5	0.25	1.54	2.0
10	CF7	Delrin	0.25	0.375	1.54	0.80
11	CF8	Delrin	0.50	0.375	1.54	1.25
12	CF9	Delrin	0.75	0.375	1.54	2.0
13	CF10	Teflon	0.375	0.25	2.18	1.25
14	CF11	Teflon	0.50	0.25	2.18	2.0
15	CF12	Nylon	0.50	0.50	1.15	1.0
16	CF13	Nylon	0.75	0.50	1.15	1.50
17	CF14	Nylon	0.875	0.50	1.15	1.50
18	CF15	Nylon	0.375	0.50	1.15	0.75

Table 2
Liquids used in sedimentation experiments

Liquid	Density (g/mL)	Mol. weight[a] (g s/mol)	Viscosity (η_0) (Pa s)	Relaxation time (s)
PAA-AN934SH (0.56%)	1.1	$(14-17)\times 10^6$	90.067	0.285

[a]Information obtained from SNF Inc.

Table 3
Observation of terminal angle

Number	Name	Height (in)	Observation time	Initial angle (deg)	Terminal angle (deg)
1	E1	33.5	3 h 33 min	0	90
2	E2	33.5	9 s	0	90
3	CR1	33.5	1 min 55 s	0	90
4	CF1	31	1 h 12 min	7	0
5	CF2	31	56 min	9	90
6	CF3	31	55 min	0	90
7	CF4	31	32 min	10	90
8	CF5	31	30 min	0	90
9	CF6	33.5	1 h 15 min	0	90
10	CF7	31	4 h	10	0
11	CF8	31	2 h 30 min	0	90
12	CF9	31	1 h 4 min	0	90
13	CF10	31	5 min 35 s	0	63.5
14	CF11	31	3 min	0	51.4
15	CF12	21	3 h 51 min	−13	44
16	CF13	29.4	3 h 50 min	11	93
17	CF14	17	2 h 33 min	10	90
18	CF15	20.5	4 h 20 min	8	0

In the table, we note the terminal orientations assumed by the different particles and also the fall times. The remaining observations can be subdivided into particles with γ, either equal to, less than or greater than 1. We note that for particles where $\gamma > 1$, the cylinders have a terminal orientation of 90°, while in the case of particles with $\gamma < 1$, the terminal orientation is 0°. However, when $\gamma = 1$, the particles CF1 and CF12 assume the shape-tilting orientation. We certainly see a dependence of the terminal orientation upon the ratio γ. However, this dependence is not continuous. Instead, there is a discontinuity at $\gamma = 1$ as θ flips from 0° to the shape-tilting orientation, to 90°.

In the case of the particles CF10 and CF11, the relatively higher density of the particles, results in a short travel time of the order of a few minutes. In this time-span, the particles assume an orientation with their edges aligned, namely the shape-tilting position. These two observations would seem to be compatible with the earlier work in the literature. However, we argue, based on our other observations that the shape-tilting phenomenon is observed due to the short fall time. If the particles were allowed to sediment for longer times, the orientation is very likely to change to a vertical position.

With observation times over 5 h, as in the case of particle CF12, there is little doubt that we are indeed in the Stokes regime and that the orientation that we record is indicative of the terminal state of the falling cylinder. In Figs. 2 and 3, we indicate the transition of the orientation angle with time for the particles, CF1–CF5 and CF12–CF14. These

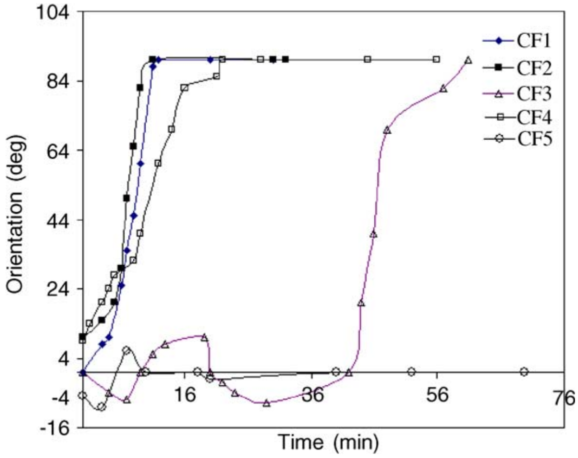

Fig. 2. Transition in orientation of PET particles CF1–CF5.

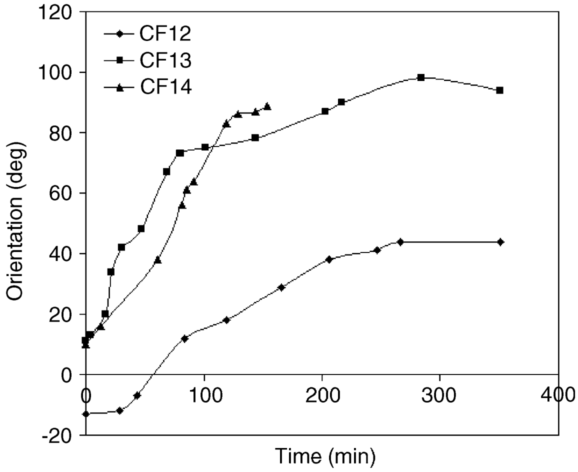

Fig. 3. Transition in orientation of nylon particles CF12–CF14.

particles also indicate the impact of the parameter γ upon this behavior, leading to the obvious conclusion that for $\gamma \neq 1$, shape-tilting is indeed a transient state.

4. A symmetry argument

In some earlier papers (Vaidya, 2004b, 2005), it has been shown that the inherent geometric symmetry of the sedimenting body is the key to understanding the terminal orientations that the body can attain in its steady state. Our analysis of the possible terminal orientations of rigid bodies of different shapes, free-falling in a second-order fluid reveals that there are as many terminal states as the number of simultaneous symmetry planes that the body possesses. We may therefore employ the argument originated in Happel and Brenner (1965) for the Stokes flow regime and replicated in Vaidya (2004b, 2005) to the case here, to try and understand the shape-tilting phenomenon for a cylinder of varying aspect ratios.

The entire argument, due to its mathematical complexity, will not be repeated here. The terminal orientation of the body is obtained from the realization that in its steady state, the net torque acting upon the body must be zero. Furthermore, imposing symmetries of the sedimenting bodies we are able to analyze the orientations that bodies of different shapes can assume. The argument is, however, mainly possible due to the assumption of zero Reynolds number, which allows us to invoke the symmetries of the flow. We argue later in this section that the assumptions made in our analysis are completely valid and guarantee us results, which are very similar to those observed in experiments. For the details, the reader is referred to the references cited above. Firstly, we define the body with *fore-aft symmetry* as one which possesses an axis of revolution and in addition also two planes of mirror symmetry. It is then easy to see that a cylinder is an example of such a body. Additionally, in the special case when $\gamma = 1$, the cylinder contains two additional symmetry planes, namely the planes bisecting the opposite edges. For cylinders with $\gamma \neq 1$, this particular symmetry does not exist. This point is elucidated in Fig. 4 in two dimensions, without loss of generality.

In light of this fact, it can be immediately seen that our mathematical argument will permit only those cylinders with $\gamma = 1$ to exhibit the shape-tilting orientation whereas, in the case of cylinders with $\gamma \neq 1$, shape-tilting cannot occur since it lacks the required additional symmetry. Therefore, cylinders with $\gamma = 1$ can fall, in their terminal states, either along the axis of revolution, perpendicular to the axis of revolution or in an edge-to-edge orientation. However, cylinders with $\gamma \neq 1$ can only orient themselves the first two ways. Therefore, this reinforces our argument that the shape-tilting orientation observed in the previous literature for cylinders with $\gamma \neq 1$ is very likely to be a transient phenomenon.

The choice of the second-order fluid model as a basis for our analysis is made firstly because it is the simplest example of a non-Newtonian fluid and secondly because, when setting Reynolds number to zero, it allows for a simpler, symmetry argument which cannot be performed upon any other model. The choice of this model is further justified because experiments on terminal orientations indicate that viscosity plays no role in the orientation behavior (Leal, 1975). Whereas our symmetry-based analysis indicates that up to three different terminal orientations are *attainable*, it does not tell us which of these three are *stable*. The varying orientations may be qualitatively attributed to the effect of inertial versus the viscoelastic torques, upon the falling cylinder. Therefore, accounting for both the inertial and viscoelastic content of the polymer in our model would allow us the added benefit of understanding the nature of stability of the attainable solutions (Galdi and Vaidya, 2001; Galdi et al., 2002), but does not add any additional information about the attainability of the solutions. However, at this stage, stability still remains an open question.

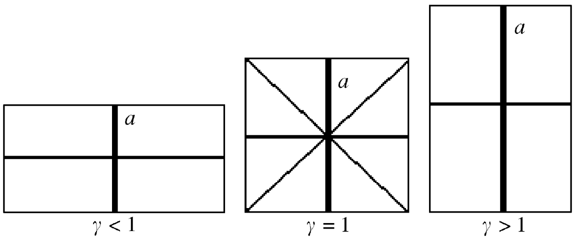

Fig. 4. Symmetries of cylinders with varying values of γ, where a is the axis of revolution.

5. Discussion

In conclusion, we make the following observations. (1) The orientation behavior of bodies with rounded ends is different from that of those with flat ends. We note that the particles E1, E2 and CR1 tend to turn and reach their terminal states much faster than the remaining particles (with flat ends) used in our study. While the former set of particles reach their steady state in a matter of minutes, the latter set of particles can take up to several hours for the same. This fact is also strengthened by observations made on sedimentation of ellipsoids and round-ended cylinders in methylcellulose polymer (Vaidya, 2004a) where the observation time is of the order of a few minutes at most. (2) Even for flat-ended cylinders, there is variation in the orientation behavior of the particles depending upon their aspect ratios (namely γ). The terminal orientation of cylinders varies and can take on either 0° or 90° for $\gamma < 1$ or $\gamma > 1$, respectively. In the special case when $\gamma = 1$ the particle tends to fall with its opposite edges aligned along the direction of gravity, that is to say, it adopts the shape-tilting orientation. Our particular contribution in this paper has been to establish that, as observed in previous literature, shape-tilting does not occur for all cylinders but only for those with $\gamma = 1$. In fact, previous experiments are marred by lack of sufficient observation time which results in an erred inference. (3) We see that the set of attainable solutions obtained from the theoretical investigations of Vaidya (2004b, 2005) are in perfect agreement with experimental observations.

In fact, combining the results of the theoretical and experimental observations, we argue that (a) the sedimenting cylinder can take on as many terminal states as the number of symmetry planes and (b) the stable steady orientation is along the longest axis among the set of attainable solutions. Therefore, in the case of a flat-ended cylinder, based on Fig. 4, the particle can fall either along the axis of revolution, a, or perpendicular to it for $\gamma < 1$ and $\gamma > 1$. However, for $\gamma < 1$, the stable state is in the direction perpendicular to a, while for $\gamma > 1$ the stable state is along a. When $\gamma = 1$, the longest axis is the one for which the cylinder shape-tilts. At this stage, the inference (b) made above, remains a conjecture but is consistent with our experiments.

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